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Abstract- The SonTek® FlowTracker is an acoustic Doppler velocimeter (ADV) designed for wading discharge measurements using established methodology (ISO, U.S. Geological Survey, and others). There is increasing interest and emphasis on the uncertainty of hydrographic measurements, including wading discharge measurements. Several sources (including ISO standards) have developed algorithms for calculating this uncertainty. To date, these procedures have been used primarily as research and post-processing tools, and have had limited direct impact on field measurement techniques. Two different uncertainty calculations have recently been implemented in the FlowTracker: the ISO calculation and one developed by researchers at the U.S. Geological Survey. The algorithms calculate the overall uncertainty of the discharge measurement and the contribution of different factors (depth, velocity, etc.). The calculations are performed in real time, providing the operator with immediate feedback on measurement uncertainty and the components that contribute to the uncertainty. The details of both uncertainty calculations are described, and results of each calculation are compared for a number of field measurements.

I. BACKGROUND

The SonTek® FlowTracker is acoustic Doppler velocimeter (ADV)^[1] designed for wading discharge measurements^{[2][3][4]}. It includes algorithms for the measurement and calculation of discharge following established methodology (including ISO and U.S. Geological Survey standards). The FlowTracker was introduced in 2001 and has been adopted for use world wide (over 1000 systems sold to date). A common FlowTracker mounting, showing the probe and handheld controller on a top setting wading rod, is shown in Figure 1.



Figure 1 – SonTek FlowTracker on Top Setting Wading Rod

II. OVERVIEW

Beginning with firmware version 3.0 and software version 2.00, the FlowTracker estimates the uncertainty of every discharge measurement. This calculation is done

two different ways: the ISO calculation and a method referred to as the Statistical calculation.

The ISO uncertainty calculation is based upon the international standard and provides users with the results of a published, accepted technique. However, in some cases this calculation does not provide a reliable indicator of data quality.

The Statistical uncertainty calculation uses a method developed by researchers at the U.S. Geological Survey. This is the default calculation used by the FlowTracker as it appears to provide a more reliable indicator of measurement quality.

In the FlowTracker real time display, the user can select which discharge uncertainty calculation to display. The FlowTracker software displays the results of both uncertainty calculations.

III. ISO CALCULATION

The FlowTracker implementation of the ISO uncertainty calculation is based upon a working version of ISO standard number 748^[5] from 2003. While it is normally not appropriate to use a working version, an exception was made since the working version provides a more thorough calculation than the released ISO standard (dated 1997).

Equation 1 shows the ISO method to calculate uncertainty applied to a FlowTracker discharge measurement. All values are given as relative (percentage) uncertainty.

Equation 1 – ISO Uncertainty Calculation

$$u_Q^2 = u_m^2 + u_s^2 + \frac{\sum_{i=1}^m \left((b_i d_i v_i)^2 \left(u_{b_i}^2 + u_{d_i}^2 + u_{v_i}^2 + \left(\frac{u_{c_i}^2 + u_{e_i}^2}{n_i} \right) \right) \right)}{\left(\sum_{i=1}^m (b_i d_i v_i) \right)^2}$$

- u_Q = uncertainty in discharge
- u_m = uncertainty due to number of verticals (see below)
- u_s = uncertainty due to calibration errors in measurements of width, depth and velocity. This is assumed to be dominated by accuracy of the FlowTracker calibration (1%).
- m = number of verticals across the width of the stream
- b_i = width at vertical i
- d_i = depth at vertical i

- v_i = mean velocity at vertical i
- u_{bi} = uncertainty in the width measurement at vertical i . This is assumed to be 0.5%.
- u_{di} = uncertainty in the depth measurement at vertical i . This is assumed to be 0.5% for depth > 0.30 m (1 ft), and 1.5% for depth < 0.30 m (1 ft).
- u_{pi} = uncertainty due to the limited number of velocity measurements at vertical i (see below)
- $u_{ci} + u_{ei}$ = uncertainty in velocity measurements at vertical i , with contributions from instrument uncertainty (u_{ci}) and real fluctuations in the river velocity (u_{ei}). The combination of these two terms is directly measured by the FlowTracker as the standard error of velocity (V_{Lerr}), and is calculated as $(u_{ci}^2 + u_{ei}^2 = (V_{Lerr} / v_i)^2)$
- n_i = the number of velocity measurements at vertical i

Velocity and depth are measured at a limited number of verticals across the stream, and are assumed to vary linearly between them. To estimate the uncertainty of this assumption, the ISO provides a guideline based upon the number of verticals shown in Table 1.

Table 1 – ISO Uncertainty for Number of Verticals

Number of Verticals	Uncertainty % (u_m)
5	7.5
10	4.5
15	3.0
20	2.5
25	2.0
30	1.5
35	1.0
40	1.0
45	1.0

Sauer and Meyer^[6] provide essentially the same data, and convert this to Equation 2 to calculate this uncertainty for any number of verticals (u_m is in percent; m is the number of verticals). This is the equation used by the FlowTracker when calculating the ISO uncertainty estimate.

Equation 2 – ISO Uncertainty for Number of Verticals

$$u_m = 32 * m^{-0.88}$$

This estimate is based on a statistical analysis of many rivers. It does not take into account the data available at an individual site which could strongly influence the overall uncertainty. For example, it might be possible with 5 verticals to accurately measure the flow in a broad concrete channel of constant depth, as the velocity distribution will likely be very consistent. In comparison, a natural stream can show large velocity and depth changes and the accuracy of a discharge measurement with 5 verticals would be much lower. The ISO calculation does not account for this difference. This is perhaps the most significant shortcoming of the ISO calculation.

A limited number of velocity measurements are made at each vertical; the mean velocity is calculated using as-

sumptions about the velocity distribution. The ISO standard provides the data in Table 2 to estimate the uncertainty associated with these assumptions.

Table 2 – ISO Uncertainty for Number of Velocity Measurements

Measurement Method	Uncertainty (u_{pi})
1 point (0.6 * depth)	7.5%
2 points (0.2 and 0.8 * depth)	3.5%
5 points (surface, 0.2 / 0.6 / 0.8 * depth, bottom)	2.5%
Distribution method (change between points < 20%)	0.5%

For the FlowTracker, we have simplified Table 2 to estimate the uncertainty based only on the number of measurements in the vertical as shown in Table 3.

Table 3 – SonTek Formulation of ISO Uncertainty For Number of Velocity Measurements

Number of Measurements	Uncertainty (u_{pi})
1	7.5%
2	3.5%
3	3.0%
4	2.7%
5 or more	2.5%

In Equation 1, the ISO calculation breaks the sources of uncertainty into two groups. The first group are uncertainty sources that are applied for each vertical: width (u_{wi}), depth (u_{di}), method (u_{pi} , for the number of velocity measurements at each vertical), and velocity ($u_{ci} + u_{ei}$). These uncertainty sources are weighted based on the discharge of each vertical. The second group contains values applied to the measurement as a whole: the accuracy of instrument calibration (u_c), and the number of verticals (u_m). All uncertainty sources are assumed to be independent.

Although Equation 1 appears complicated at first glance, it is straight forward to implement in the FlowTracker. Each term is either measured directly by the FlowTracker or can be determined from the ISO standard. The summation to determine uncertainty is done by the FlowTracker at the same time as the discharge calculation (which uses a similar summation).

In addition to overall uncertainty, the FlowTracker looks at the contribution of each parameter. To calculate the contribution of each parameter, the calculation is repeated while setting all other parameters to zero. At the end of each discharge measurement, the FlowTracker real time display shows the overall uncertainty and the largest individual source of uncertainty. The FlowTracker software shows the contribution of each parameter.

- Accuracy (u_c): uncertainty due to the accuracy of the FlowTracker calibration
- Depth (u_{di}): uncertainty due to depth measurements

- Method (u_{pl}): uncertainty due to the number and location of velocity measurements at each vertical
- Number of verticals (u_m): uncertainty due to a limited number of verticals
- Velocity ($u_{ci} + u_{ei}$): uncertainty due to velocity measurements (instrument uncertainty and real fluctuations in the flow)
- Width (u_w): uncertainty due to width measurements

IV. STATISTICAL CALCULATION

The method we refer to as the Statistical calculation was developed by researchers at the U.S. Geological Survey (USGS): Tim Cohn, Julie Kiang, and Robert Mason^[7]. It has also been called the interpolated difference technique, although a final name has not been selected. As of August 2006, they have not published this technique but have plans to do so in the future. The calculation described here should be considered preliminary, and may be subject to change.

The Statistical technique takes a very different approach from the ISO method. The ISO looks at the physical characteristics of the measurement and discharge calculation to estimate uncertainty. The Statistical technique is a strictly statistical approach, using adjacent values of each measured variable to estimate the uncertainty in these measurements. This paper presents only an overview of this technique, deferring a full description to future publications of Cohn, Kiang and Mason.

The basic form of the Statistical calculation (Equation 3) is similar to the ISO calculation. As with the ISO calculation, all values in Equation 3 are given as relative (percentage) uncertainty.

Equation 3 – Statistical Uncertainty Calculation

$$u_Q^2 = u_s^2 + \frac{\sum_{i=1}^m \left((b_i d_i v_i)^2 (u_{bi}^2 + u_{di}^2 + u_{vi}^2) \right)}{\left(\sum_{i=1}^m (b_i d_i v_i) \right)^2}$$

- u_Q = uncertainty in discharge
- u_s = uncertainty due to calibration errors in measurements of width, depth and velocity. This is assumed to be dominated by accuracy of the FlowTracker calibration (1%).
- m = number of verticals across the width of the stream
- b_i = width at vertical i
- d_i = depth at vertical i
- v_i = mean velocity at vertical i
- u_{bi} = uncertainty in width at vertical i . The Statistical technique does not include a method for calculating this value, so we use the ISO value of 0.5%.
- u_{di} = uncertainty in depth at vertical i (see below).
- u_{vi} = uncertainty in velocity at vertical i (see below).

To estimate the uncertainty in depth and velocity, the Statistical technique uses adjacent measurements. The

calculation is the same for depth or velocity (the depth calculation is shown here).

A basic assumption of a discharge measurement is that velocity and depth change linearly between verticals. Following this assumption, we can estimate the depth at vertical i (d_i) by using depth values from the adjacent verticals (d_{i-1} and d_{i+1}). For simplicity the calculation below assumes equal spacing of verticals; the FlowTracker uses a linear interpolation based on the location of each vertical for the estimated value.

$$d_{i_est} = (d_{i-1} + d_{i+1}) / 2$$

An estimate of the uncertainty in depth for vertical i can be calculated as the difference between the estimated and measured depth.

$$\Delta_i = d_{i_est} - d_i$$

Individual uncertainty estimates (Δ_i) are subject to considerable variability; combining all estimates from a given measurement gives a better overall estimate of uncertainty. Equation 4 calculates an overall estimated of the uncertainty in depth measurements (σ_d), a statistical average of the individual uncertainty estimates (Δ_i). This value (σ_d) is in depth units (m or ft). (The derivation of Equation 4 is deferred to future publications of Cohn, Kiang and Mason.)

Equation 4 – Statistical Depth Uncertainty (Depth Units)

$$\sigma_d^2 = \left(\frac{2}{3} \right) \left(\frac{1}{(m-2)} \right) \sum_2^{m-1} (\Delta_i^2)$$

The relative uncertainty is then calculated in Equation 5. This relative depth uncertainty (u_{di}) is used directly in Equation 3. A similar term is calculated for velocity (u_{vi}).

Equation 5 – Statistical Depth Uncertainty (Relative)

$$u_{di} = \left(\frac{\sigma_d}{d_i} \right)$$

Perhaps the biggest advantage of the Statistical technique is that the estimated uncertainty takes into account variability in depth and velocity across the stream, and hence includes measurement uncertainty, stream conditions (i.e. different bottom types), and the assumption that depth and velocity change linearly between stations.

As with the ISO calculation, Equation 3 breaks the sources of uncertainty into two groups. The first are uncertainty sources that are applied for each vertical: width (u_{wi}), depth (u_{di}), and velocity (u_{vi}). These uncertainty sources are weighted based on the discharge of each vertical. The other uncertainty source is applied to the measurement as a whole: the accuracy of instrument calibration (u_s). All uncertainty sources are assumed to be independent.

In addition to overall uncertainty, the FlowTracker looks at the contribution of each parameter. To calculate the contribution of each parameter, the calculation is

repeated while setting all other parameters to 0. At the end of each discharge measurement, the FlowTracker real time display shows overall uncertainty and the largest individual source of uncertainty. The FlowTracker software shows the contribution of each parameter.

- Accuracy (u_a): uncertainty due to the accuracy of FlowTracker calibration.
- Depth (u_d): this term includes both uncertainty in the depth measurement and the effect of changes in depth between verticals.
- Velocity (u_v): this term includes instrument uncertainty, real variations in velocity (turbulence), and the effect of changes in velocity between verticals.
- Width (u_w): uncertainty due to width measurements

V. COMPARISON

Why offer two different uncertainty calculations - shouldn't one be sufficient? To answer this, we look at the results of each method.

The ISO calculation seems a natural choice: it is well documented and from an internationally recognized agency. However, analysis shows the ISO does not always provide a meaningful indication of the measurement quality. In contrast, the Statistical technique appears to provide a good indicator of measurement quality, particularly at sites with variable flow conditions. However, it is currently an unpublished technique and may be subject to change in the future. Since there are drawbacks to each technique, we decided to present results from both calculations.

To compare the two uncertainty calculations, we used a set of 24 FlowTracker discharge measurements. These represent a range of conditions: discharge values from 0.004 to 8.6 m³/s (0.13 to 300 ft³/s) and mean velocity from 0.01 to 0.50 m/s (0.03 to 1.6 ft/s). The measurements were all made in natural streams at a variety of locations in North America. Figure 2 compares the Statistical and ISO calculations from all 24 files.

- The Statistical calculation shows uncertainty values from 2.1 to 19%; the ISO calculation shows values from 2.4 to 8.4%.
- If you remove one outlier (a file with very low velocity), the Statistical calculation varies from 2.1 to 15.1% while the ISO varies only from 2.4 to 4.3%.
- Uncertainty under 5% is considered a "Good" measurement by many agencies; hence the ISO equation would rate all but one of these measurements as "Good". This is clearly not the case upon closer analysis of some files.

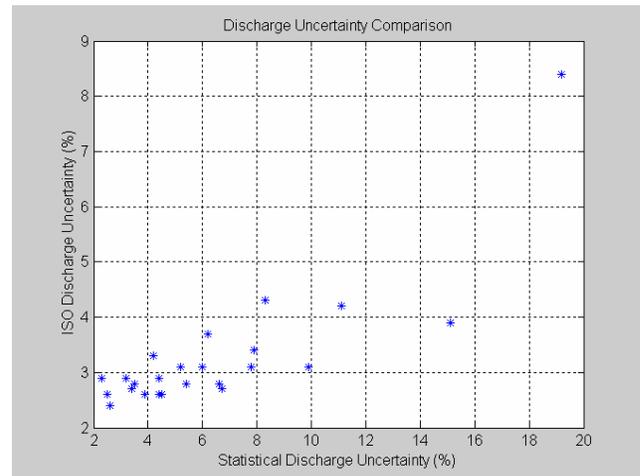


Figure 2 – Uncertainty Calculation Comparison

To understand the differences, we look at some individual files. Figure 3 shows depth and velocity profiles from a site where Statistical uncertainty is 2.5% while ISO uncertainty is 2.6%. As both calculations indicate, this is a good measurement with smooth, linear variations in depth and velocity with few large inconsistencies. Both calculations correctly represent this.

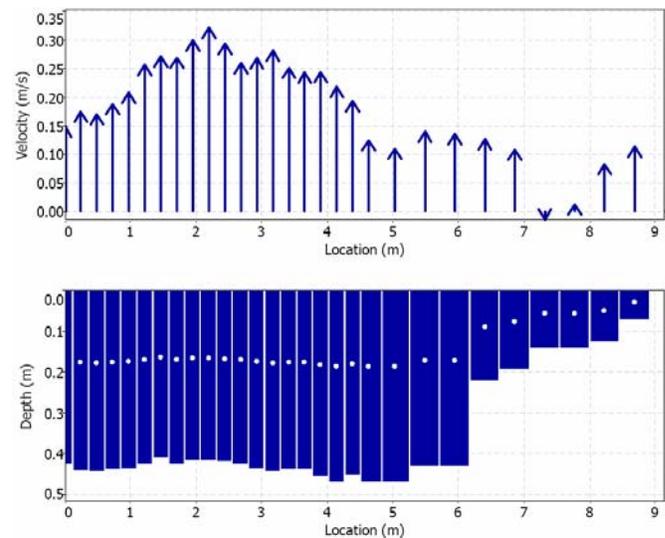


Figure 3 – Uncertainty Comparison, "Good" File

Figure 4 shows depth and velocity profiles from a file where the Statistical uncertainty is 15.1% while the ISO uncertainty is 3.9%. Looking closely at the measurement, there are a number of large and dramatic changes in both depth and velocity (particularly velocity, for example measurements at locations 5.5 and 8.1 m). This indicates either unusual flow conditions (which would require more verticals to resolve) or measurement problems. The ISO calculation still reports an uncertainty (3.9%) that would be considered good by most users. The Statistical calculation reports a much higher uncertainty (15.2%), correctly indicating that there are areas for concern in the measurement quality.

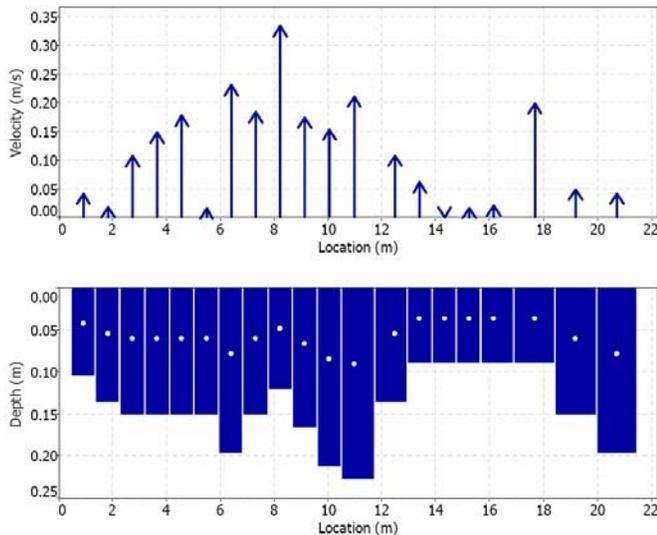


Figure 4 – Uncertainty Comparison, “Bad” File

It is also interesting to look at the contribution of each parameter to the estimated uncertainty. For the ISO calculation, 6 different parameters contribute to the overall uncertainty: width, depth, velocity, method, number of stations, and accuracy (FlowTracker calibration). For the Statistical calculation, there are 4 parameters: width, depth, velocity and accuracy (FlowTracker calibration again).

For the ISO calculation, the number of stations is the largest single component of uncertainty for 22 out of 24 files; method and velocity are each the largest source in one file. Since the number of stations parameter is essentially based on a statistical analysis of many rivers, rather than data from the specific measurement site, this raises significant concerns if it is the largest source of uncertainty. The contribution of velocity is generally small, except in cases where the mean velocity is very low (velocity is the largest component of uncertainty in a file where the mean velocity is 0.01 m/s (0.04 ft/s)). The measurement method is generally a modest contributor to overall uncertainty, but can be significant in files with low overall uncertainty (<3%). The contribution of width, depth and accuracy to the overall ISO uncertainty is small to negligible.

For the Statistical calculation, the velocity term is the largest individual source of uncertainty in all 24 files. Keep in mind that this term includes not only uncertainty in the velocity measurement, but also variation in velocity between stations (which is typically the dominating factor). Depth adds a small but notable amount to the Statistical uncertainty calculation; again, this is dominated by the variation in depth between stations. The contributions of width and accuracy are small to negligible. Analysis of this data tends to indicate that variation between stations, both of depth and velocity, are the most important factor in overall measurement uncertainty.

VI. CONCLUSIONS

The ISO and Statistical calculations provide practical methods to estimate discharge uncertainty, and have been implemented for automatic analysis in the FlowTracker. Shortcomings in the ISO calculation reduce its ability to reflect the quality of a discharge measurement; however we felt that it was still necessary to show the results of this method since it is a standard technique. Because of the ability of the Statistical calculation to better distinguish data quality, we recommend using this calculation.

With the automatic calculation of discharge uncertainty, we hope to accomplish two things: to provide operators with feedback that improves the quality of their measurements, and to contribute to data analysis that improves uncertainty calculations in the future.

Regardless of the instrument used, the quality of any field measurement relies heavily on the technique employed by the operator. One of the best ways to improve measurement quality is to provide information and feedback that helps the operator improve their technique. The FlowTracker uncertainty calculation is one part of SonTek/YSI’s efforts to provide this feedback.

VII. ACKNOWLEDGMENTS

The authors would like to thank Julie Kiang, Tim Cohn, and Mike Rehmel of the U.S. Geological Survey for their help explaining and describing the Statistical technique, and their willingness to share their work.

VIII. REFERENCES

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SonTek/YSI, founded in 1992 and advancing environmental science in over 100 countries, manufactures affordable, reliable acoustic Doppler instruments for water velocity measurement in oceans, rivers, lakes, harbors, estuaries, and laboratories. Headquarters are located in San Diego, California. Additional information can be found at www.sontek.com